Nucleon form factors in a light-front framework:
a model with instanton-induced forces

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Abstract

Nucleon form factors are evaluated in the spacelike region in a light-front framework. Our phenomenological constituent quark model is based on a relativistic formulation of the Hamiltonian dynamics. The baryon dynamics is solved in the nucleon rest frame with relativistic kinetic energy, linearly rising confining potential and a residual interaction based on instanton-induced forces, the so-called 't Hooft interaction. The wave funtions of the model are used to compute the nucleon form factors in the impulse approximation. In spite of taking no phenomenological constituent quark form factor by the one-body electromagnetic current, we obtain a very good description of all electromagnetic form factors for momentum transfers up to -3 GeV^2. The effect of the 't Hooft interaction is carefully examined in the model.

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I. INTRODUCTION

The investigation of electromagnetic form factors has doubtless a crucial importance to gain qualitative insight into the hadronic structure. Recently, the nucleon electromagnetic form factors have received much theoretical attention due to new experimental results [1-3] and several proposed experiments, which shall be realized in near future [4].

Among the most recent relativistic constituent quark models, we should high light the great achievements of the semibosonized Nambu-Jona-Lasinio model [5-7], the quark-diquark instantaneous Bethe-Salpeter (BS) framework [8], the light-front framework with simple nucleon wave funtions [9,10] and the light-front relativistic Hamiltonian dynamics (RHD) [11,12].

In the refered investigations within the RHD, the eigenvalue equation for the mass operator has been identified with the Capstick-Isgur Hamiltonian [13], which contains an interaction term composed of a confining part and a one-gluon-exchange (OGE) part. We will also use the light-front RHD [14], but our principal aim throughout this paper is to examine the influence of instanton-induced forces on the nucleon form factors.

Instanton effects have been computed by 't Hooft and others [15-17] in the soft-QCD regime. Those effects yields a residual flavour-paring force, the so-called 't Hooft interaction, which leads to good results for meson and baryon mass spectra within a nonrelativistic constituent quark model [18]. A covariant BS framework for mesons, which also takes the 't Hooft interaction into account has been investigated recently by Münz [19]. In the treatment of Münz the BS equation is formulated in the instantaneous approximation (Salpeter equation), and a remarkable success is achieved by the description of the meson spectrum and dynamical observables. To our knowledge, till now there is no relativistic treatment of nucleon form factors which includes the 't Hooft interaction in the baryon dynamics. In this sense the investigations presented here seems quite worthwhile.

The case of baryons within the BS framework is technically and also conceptually much more involved than mesons. Nevertheless, we show in the next section that under some

assumptions one can arrive at a reduced Salpeter equation for baryons, which has the same form as the eigenvalue equation of the mass operator obtained in the RHD. Therefore, we are able to identify the mass operator of the RHD with the Salpeter Hamiltonian, and the 't Hooft interaction used in the Salpeter framework can be extended to investigate baryons within the RHD.

This paper is organized as follows: in Sec. II the eigenvalue equation of the mass operator in the RHD [14] is shown to be connected to a reduced Salpeter equation, which is derived under the assumptions of free quark propagators and collective instantaneous interaction kernel. In this sense, we identify the mass operator with the Salpeter Hamiltonian with a linearly rising confining potential and a residual 't Hooft-type interaction.

The nucleon form factors are computed in Sec. III with the same formalism described by Capstick and Keister [12] to investigate the baryon electromagnetic current in the impulse approximation. Our calculations are performed in the spacelike region up to momentum transfer $-q^2 = 3 \, \mathrm{GeV}^2$ and with light-front wave funtions expanded up to the sixth harmonic-oscillator quanta. We obtain a remarkable description for the proton electric form factor G_E^p in this region. The other nucleon form factors are shown to be in reasonable agreement with the experimental data. We also examine the role of the 't Hooft interaction in the model. The configuration mixing generated by this interaction is shown to be particularly important by the description of the neutron electric form factor G_E^n.

A summary is given in Sec. IV, where we also point out possible improvements and other applications of the model.

II. THE MODEL

A. The eigenvalue equation of the mass operator

In the relativistic Hamiltonian formulation from Keister and Polyzou [14] the dynamics is solved by diagonalizing the eigenvalue equation of the mass operator, which reads in the

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case of baryons like

$$M \left| Mj\tilde{\mathbf{P}}\mu \right\rangle = (M_0 + V) \left| Mj\tilde{\mathbf{P}}\mu \right\rangle, \tag{1}$$

where $M_0 = \sum_{i=1}^{3} \omega_i = \sum_{i=1}^{3} \left(\sqrt{\mathbf{k}_i^2 + m_i^2} \right)$ is the kinetic energy operator, $\mathbf{k}_i$ and m_i are the constituent quark momentum and mass respectively, V is the interaction that fulfills the conditions of the Bakamijan-Thomas construction [20], and $\left| Mj\tilde{\mathbf{P}}\mu \right\rangle$ is the eigenstate of the mass M, total spin j, total momentum $\tilde{\mathbf{P}} = (P^+, \mathbf{P}_\perp)$ and total spin projection μ operators. Throughout this paper the same notation as in Ref. [14], with light-front coordinates $p^- = p^3 - p^0$, $\mathbf{p}_\perp = (p^1, p^2)$ and $p^+ = p^3 + p^0$, is used.

We denote here the irreducible states of the free three-particle Hilbert space by $\left| [a](\mathbf{k}_\xi, \mathbf{k}_\eta)j\tilde{\mathbf{P}}\mu \right\rangle$, where $[a]$ is the set of internal spins and angular momenta with a given coupling scheme, and $(\mathbf{k}_\xi, \mathbf{k}_\eta)$ are the relative momenta. This space is suitable to represent the above eigenvalue equation, because its elements are also eigenstates of P, j and μ.

The Bakamjian-Thomas construction yields, in the irreducible basis, the following condition for the interaction V [14]

$$\left\langle [a'](\mathbf{k}'_\xi, \mathbf{k}'_\eta)j'\tilde{\mathbf{P}}'\mu' \right| V \left| [a](\mathbf{k}_\xi, \mathbf{k}_\eta)j\tilde{\mathbf{P}}\mu \right\rangle = \delta_{j'j}\delta_{\mu'\mu}(2\pi)^3\delta(\tilde{\mathbf{P}}' - \tilde{\mathbf{P}})$$
$$\times \left\langle [a'](\mathbf{k}'_\xi, \mathbf{k}'_\eta) \left\| V^j \right\| [a](\mathbf{k}_\xi, \mathbf{k}_\eta) \right\rangle. \tag{2}$$

Therefore, the eigenvalue equation for the mass operator (1) can be put in the following form

$$\left(\left(\omega_1 + \omega_2 + \omega_3 + V^j \right) \Psi_M^j \right) ([a](\mathbf{k}_\xi, \mathbf{k}_\eta)) = M\Psi_M^j([a](\mathbf{k}_\xi, \mathbf{k}_\eta)), \tag{3}$$

where the wave funtion Ψ_M^j is defined through

$$\left\langle [a](\mathbf{k}_\xi, \mathbf{k}_\eta)j'\tilde{\mathbf{P}}'\mu' \left| Mj\tilde{\mathbf{P}}\mu \right\rangle = \delta_{j'j}\delta_{\mu'\mu}(2\pi)^3\delta(\tilde{\mathbf{P}}' - \tilde{\mathbf{P}})\Psi_M^j([a](\mathbf{k}_\xi, \mathbf{k}_\eta)). \tag{4}$$

This formulation of the RHD introduces some restrictions on the interaction, but by no means determines the explicit form of V. As we are particularly interested in the study

of instanton effects, we prefer to identify our interactions with the Salpeter Hamiltonian, because the good results achieved in the case of mesons [19].

B. The reduced Salpeter equation

A three-quark bound state with total momentum P is described in the relativistic quantum field theory by the BS amplitudes [21,22]

$$\chi_{P\alpha_1\alpha_2\alpha_3}(x_1, x_2, x_3) = \langle 0 | T\psi_{\alpha_1}(x_1)\psi_{\alpha_2}(x_2)\psi_{\alpha_3}(x_3) | P \rangle, \tag{5}$$

$$\bar{\chi}_{P\alpha_1\alpha_2\alpha_3}(x_1, x_2, x_3) = \langle P | T\bar{\psi}_{\alpha_1}(x_1)\bar{\psi}_{\alpha_2}(x_2)\bar{\psi}_{\alpha_3}(x_3) | 0 \rangle, \tag{6}$$

where $\psi_{\alpha_i}(x_i)$ is the field operator for the particle i with Dirac, flavor and color indices labeled by α_i, and T is the time ordering operator.

The amplitude $\chi_{P\alpha_1\alpha_2\alpha_3}$ fulfills in momentum space the following BS equation [22]

$$\chi_P(k_\xi, k_\eta) = S_1^F(k_1)S_2^F(k_2)S_3^F(k_3) \int \frac{d^4k_\xi'}{(2\pi)^4} \frac{d^4k_\eta'}{(2\pi)^4}(-i)K(P; k_\xi, k_\eta; k_\xi', k_\eta')\chi_P(k_\xi', k_\eta'), \tag{7}$$

where $S_i^F(k_i)$ is the full fermion propagator of particle i, and K is the BS interaction kernel. The spinor indices have been suppressed, and we have introduced the usual Jacobi momenta to factor out the c.m. movement.

The normalization condition is obtained by considering the pole contribution of the three-body Green's function associated with the BS amplitude (5) (see, e.g. Ref. [22]), it reads

$$\int \frac{d^4k_\xi}{(2\pi)^4} \frac{d^4k_\eta}{(2\pi)^4} \int \frac{d^4k_\xi'}{(2\pi)^4} \frac{d^4k_\eta'}{(2\pi)^4}\bar{\chi}_P(k_\xi, k_\eta)$$
$$\times \frac{\partial}{\partial P^0}\left[I(P; k_\xi, k_\eta; k_\xi', k_\eta') + iK(P; k_\xi, k_\eta; k_\xi', k_\eta')\right]\Bigg|_{P^0=\omega_P} \chi_P(k_\xi', k_\eta') = 2i\omega_P, \tag{8}$$

with $I = (2\pi)^8\delta(k_\xi - k_\xi')\delta(k_\eta - k_\eta')[S_1^F(p_1)]^{-1}[S_2^F(p_2)]^{-1}[S_3^F(p_3)]^{-1}$.

Our reduced Salpeter equation is derived with the following assumptions:

- The full fermion propagator $S_i^F(k_i)$ is taken as the free one with only the component that propagates forward in time

$$S_i^F(k_i) \approx i \left(\frac{\Lambda_i^+(\mathbf{k}_i)}{k_i^0 - \omega_i + i\epsilon} \right) \gamma^0, \tag{9}$$

with effective constituent quark mass m_i, standard projection operators $\Lambda_i^{(+)}(\mathbf{k}_i) = (\omega_i + H_i(\mathbf{k}_i))/(2\omega_i)$, and Dirac Hamiltonian $H_i(\mathbf{k}_i) = \gamma^0(\boldsymbol{\gamma} \cdot \mathbf{k}_i + m_i)$. We felt encouraged to accept this assumption because it not only yields some technical simplifications, e.g. in the normalization condition of the Salpeter amplitudes, but it also isolates formally the kinetic energy contribution. Hence, the consideration of other features of the baryon dynamics, e.g. the confinement is left to the interaction kernel. This assumption leads to the Tamm-Dancoff approximation (TDA) of the BS equation, which is expected to be a reasonable approximation because the nucleon is not treated as a deeply bound-state.

- The interaction kernel K is considered collective and instantaneous in the baryon rest frame, i.e.

$$K(P; k_\xi, k_\eta; k_\xi', k_\eta')\big|_{P=(M,\mathbf{0})} = V(\mathbf{k}_\xi, \mathbf{k}_\eta; \mathbf{k}_\xi', \mathbf{k}_\eta'). \tag{10}$$

Since the total momentum P is a conserved quantity, we can extend the above approximation to any frame [23] simply assuming that

$$K(P; k_\xi, k_\eta; k_\xi', k_\eta') = V(k_{\xi \perp P}, k_{\lambda \perp P}; k_{\xi \perp P}', k_{\lambda \perp P}'), \tag{11}$$

with $k_{i_{\perp P}} = k_i - (Pk_i/P^2)P$.

Some important comments should be made at this point. The interaction kernel K has contributions of two- and three-body irreducible kernels. If the two-body kernels are taken instantaneous, the total interaction cannot be put in an instantaneous form, because the quark spectator is represented by an inverse propagator, which depends on the time components k_ξ^0 and k_η^0. This is shown diagrammatically in Fig. 1. Therefore, the idea of our approximation is to assume that the kernels and their respective spectators combine to build an effective instantaneous interaction. In this sense our interaction can be called collective.

For instance, a similar argument has already been used by Mitra and Santhanam [24]. Nevertheless, they use two-body instantaneous kernels and suppress the non-instantaneity due to the quark spectator by introducing appropriate δ-functions for the time-component of the relative momenta.

With the above assumptions the BS equation (7) can be integrated out over the time-component of the relative momenta. Therefore, we get a reduced Salpeter equation, which can be written as an eigenvalue equation like

$$
\begin{aligned}
(\mathcal{H}\Phi_M)(\mathbf{k}_\xi, \mathbf{k}_\eta) &= M\Phi_M(\mathbf{k}_\xi, \mathbf{k}_\eta) \\
&= (\omega_1 + \omega_2 + \omega_3)\Phi_M(\mathbf{k}_\xi, \mathbf{k}_\eta) \\
&\quad + \Lambda^{+++}(\gamma^0 \otimes \gamma^0 \otimes \gamma^0) \int \frac{d^3\mathbf{k}'_\xi}{(2\pi)^3} \frac{d^3\mathbf{k}'_\eta}{(2\pi)^3} V(\mathbf{k}_\xi, \mathbf{k}_\eta; \mathbf{k}'_\xi, \mathbf{k}'_\eta)\Phi_M(\mathbf{k}'_\xi, \mathbf{k}'_\eta),
\end{aligned} \tag{12}
$$

with the Salpeter amplitude

$$
\Phi_M(\mathbf{k}_\xi, \mathbf{k}_\eta) = \left(\int \frac{dk^0_\xi}{2\pi} \frac{dk^0_\eta}{2\pi} \chi_P(k_\xi, k_\eta) \right)_{P=(M,\mathbf{0})} \tag{13}
$$

and the tensor product of projection operators

$$
\Lambda^{+++} = \Lambda_1^+(\mathbf{k}_1) \otimes \Lambda_2^+(\mathbf{k}_2) \otimes \Lambda_3^+(\mathbf{k}_3). \tag{14}
$$

From Eq. (8), we obtain the normalization condition of the Salpeter amplitudes, which is given by

$$
\int \frac{d^3\mathbf{k}_\xi}{(2\pi)^3} \frac{d^3\mathbf{k}_\eta}{(2\pi)^3} \, \text{tr} \left\{ \Phi_M^\dagger(\mathbf{k}_\xi, \mathbf{k}_\eta)\Phi_M(\mathbf{k}_\xi, \mathbf{k}_\eta) \right\} = 2M. \tag{15}
$$

We see that the reduced Salpeter equation (12) has the same form as the eigenvalue equation of the mass operator (3). Moreover, in the present formulation the Salpeter Hamiltonian $\mathcal{H}$ is explicitly positive definite due the presence of the projection operator Λ^{+++}, and the Eq. (15) yields real eigenvalues. Therefore, the Salpeter amplitudes remain in a Hilbert space like the wave funtions in the RHD, and we can identify the mass operator with the Salpeter Hamiltonian $\mathcal{H}$.

C. The internal baryon dynamics

Like the nonrelativistic calculation, we still can examine the internal baryon dynamics with basically three components, namely the kinetic energy, the confinement and the fine-structure interactions.

Unfortunately, one is not able to derive from the QCD Lagrangian the Salpeter kernel that describes the confinement mechanism. Therefore, our confinement kernels must be phenomenologically motivated. The experimental analysis of the Regge trajectories within the Chew-Frautchi plot [25], as well as lattice calculations of QCD [26] support the assumption of a string-like behavior of confinement. The two- and three-body string potentials are given by

$$V_{\text{conf}}^{\text{2-body}} = a_2 + b_2 \sum_{i<j} |\mathbf{x}_i - \mathbf{x}_j|, \tag{16}$$

$$V_{\text{conf}}^{\text{3-body}} = a_3 + b_3 \min_{\mathbf{x}_0} \left(\sum_{i=1}^{3} |\mathbf{x}_i - \mathbf{x}_0| \right). \tag{17}$$

It is well known that the three-body potential can be well approximated by the two-body string [27]. Therefore, the scalar part of the confinement potential is parameterized in this work just with a two-body string potential. This local potential yields a convolution-type kernel in momentum space.

There is also no precise candidate for the Dirac structure of the confinement potential from pure QCD analysis. The phenomenological analysis of the Salpeter framework for mesons [19] shows that the scalar $1 \otimes 1$ spin structure yields a resonable Regge behavior, and the timelike vector $\gamma^0 \otimes \gamma^0$ spin structure reproduces the masses and decays of the low lying mesons. We make an extension of this analysis for the case of baryons taking a combination of the above structures, which reasonably describes the Regge behavior in the Δ-sector. The confinement potential is parameterized here like

$$V_{\text{conf}}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) = \sum_{(123)} [a + b(| \mathbf{x}_1 - \mathbf{x}_2 |)] \tag{18}$$

$$\times \frac{1}{2} (1 \otimes 1 \otimes 1 \otimes + \gamma^0 \otimes \gamma^0 \otimes 1). \tag{19}$$

Also in the Salpeter framework, the baryon spectrum cannot be described with a confining potential alone, and the introduction of a residual interaction is necessary. We use here the 't Hooft interaction, which is based on instanton effects extracted from the non-perturbative soft regime of the QCD [15-17]. In contrast with the OGE calculations (see e.g. Ref. [28]), the 't Hooft interaction is a flavor-dependent pairing force. It means that this interaction is able to remove, e.g., the π-η and the N-Δ degeneracies, as observed in nonrelativistic calculations [18]. Recent calculations with the 't Hooft interaction within a Salpeter framework for mesons [19] not only yield the correct π, η splitting, but also solve the $n\bar{n}-$,$s\bar{s}$-mixing for the η meson. Therefore, we felt encouraged to extend such consideration for the case of baryons.

The two-body 't Hooft Lagrangian has the following structure

$$
\begin{aligned}
\Delta\mathcal{L}(2) = &-\frac{3}{16}\sum_i\sum_{kl}\sum_{mn} g_{\text{eff}}(i)\epsilon_{ikl}\epsilon_{imn} \\
&\times\left\{: \bar{q}_k\bar{q}_l\left(\mathbf{1}\otimes\mathbf{1}+\gamma^5\otimes\gamma^5\right)\left(2\mathsf{P}_3^{\mathcal{C}}+\mathsf{P}_6^{\mathcal{C}}\right)q_mq_n :\right\},
\end{aligned}
\tag{20}
$$

where the sum are over flavor indices and the effective coupling constant g_{eff} depends on the quark flavor, and it is taken as a free parameter in our model. The tensor notation

$$
\bar{q}\bar{q}(A\otimes B)qq = \sum_{ij}\sum_{kl}\bar{q}_i\bar{q}_j A_{ik}B_{jk}q_kq_l
\tag{21}
$$

has been used for Dirac and color indices. The operators $\mathsf{P}_6^{\mathcal{C}}$ and $\mathsf{P}_3^{\mathcal{C}}$ are respectively color sextet and anti-triplet projectors. They are given by

$$
\begin{aligned}
\mathsf{P}_6^{\mathcal{C}} &:= \frac{2}{3}\mathbf{1}^{\mathcal{C}}+\frac{1}{4}\boldsymbol{\lambda}\cdot\boldsymbol{\lambda}, \\
\mathsf{P}_3^{\mathcal{C}} &:= \frac{1}{3}\mathbf{1}^{\mathcal{C}}-\frac{1}{4}\boldsymbol{\lambda}\cdot\boldsymbol{\lambda},
\end{aligned}
\tag{22}
$$

where λ^a ($a = 1, ..., 8$) are the $SU(3)$ color matrices.

The 't Hooft interaction derived using the Wick's theorem and the Lagrangian (20) is essentially a two-body interaction [19]. It is employed in the three-body Salpeter kernel taking the following three-body extension

$$V_{\text{t Hooft}}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) = \sum_{i<j} V^{ij}_{\text{t Hooft}}(\mathbf{x}_i - \mathbf{x}_j), \tag{23}$$

with

$$V^{12}_{\text{t Hooft}}(\mathbf{x}_1 - \mathbf{x}_2) = -4 \left(g \mathsf{P}^{\mathcal{F}}_{\mathcal{A}_{12}}(nn) + g' \mathsf{P}^{\mathcal{F}}_{\mathcal{A}_{12}}(ns) \right)$$
$$\times \left(\mathbf{1} \otimes \mathbf{1} \otimes \mathbf{1} + \gamma^5 \otimes \gamma^5 \otimes \mathbf{1} \right) \delta^3(\mathbf{x}_1 - \mathbf{x}_2), \tag{24}$$

where $g = \frac{3}{8} g_{\text{eff}}(n)$ and $g' = \frac{3}{8} g_{\text{eff}}(s)$ are the coupling constants. We use n to denote the u and d flavors. $\mathsf{P}^{\mathcal{F}}_{\mathcal{A}_{12}}(nn)$ and $\mathsf{P}^{\mathcal{F}}_{\mathcal{A}_{12}}(ns)$ are projection operators of flavor states that are anti-symmetric under permutation of the first and second quarks.

The point-like 't Hooft interaction is regularized with a Gaussian function like

$$\delta^3(\mathbf{x}_1 - \mathbf{x}_2) \rightarrow \frac{1}{\lambda^3 \pi^{3/2}} \exp\left(-\frac{|\mathbf{x}_1 - \mathbf{x}_2|^2}{\lambda^2} \right), \tag{25}$$

where the finite range λ is taken as a free parameter. Like the confinement term, this regularization also yields a convolution-type kernel in momentum space.

III. RESULTS AND DISCUSSION

As we aim to investigate the nucleon form factors, our analysis here will concern only the nonstrange sector. Our models contains five parameters, namely the nonstrange quark mass m_n, the constant a and the slope b of the confinement potential (19), the coupling constant g and the effective range λ of the 't Hooft interaction (23). We examine two sets of parameters with different constituent quark masses, which are shown in Tab. I.

Equation (12) has been solved by diagonalizing the Hamiltonian in a harmonic-oscillator basis up to 14 quanta and by applying the Rayleight-Ritz variational principle. The confinement parameters a and b have been fixed in such a way as to yield the best Regge behavior in the Δ-sector. We have chosen the coupling g and the effective range λ to reproduce the separation between the ground states of the nucleon and the Δ. The computed masses in Models 1 and 2 are shown in Tab. II (in MeV). Experimental data are from Ref. [29]. Both models lead to a reasonable description of the nucleon $N\frac{1}{2}^+$ ground state and of the

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Regge trajectory in the Δ-sector. The problems with the Roper resonance $N(1440)$ and with the nucleon ground state $N(1535)$ with negative parity are in fact well known from nonrelativistic constituent quark models [13, 18] and the situation here remains basically unchanged.

The component I^+ of the electromagnetic current operator I^μ contains all the information necessary to obtain the nucleon electromagnetic form factors [14]. This is one of the advantages in using light-front coordinates. The nucleon Pauli form factor $F_1(Q^2)$ and the Dirac form factor $F_2(Q^2)$ are obtained from

$$\left\langle Mj\tilde{\mathbf{P}}'\mu' \right| I^+(0) \left| Mj\tilde{\mathbf{P}}\mu \right\rangle = F_1(Q^2)\delta_{\mu'\mu} - i(\sigma_y)_{\mu'\mu}\sqrt{\frac{Q^2}{4M^2}}F_2(Q^2), \tag{26}$$

where $Q^2 = -q^2$ and M is the nucleon mass. The experimental results are usually given in terms of electric and magnetic form factors, G_E and G_M respectively, which are defined as

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2),$$
$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2).$$

To compute the electromagnetic current, we use the same light-front framework from Capstick and Keister, which is described in detail in Ref. [12]. The current matrix element between initial and final nucleon states is expanded in sets of free-particle states as

$$\left\langle M'j; \tilde{\mathbf{P}}'\mu' \right| I^+(0) \left| Mj\tilde{\mathbf{P}}\mu \right\rangle = \int \frac{d\tilde{\mathbf{p}}'_1}{(2\pi)^3}\frac{d\tilde{\mathbf{p}}'_2}{(2\pi)^3}\frac{d\tilde{\mathbf{p}}'_3}{(2\pi)^3}\int \frac{d\tilde{\mathbf{p}}_1}{(2\pi)^3}\frac{d\tilde{\mathbf{p}}_2}{(2\pi)^3}\frac{d\tilde{\mathbf{p}}_2}{(2\pi)^3}$$
$$\times \sum_{\mu_i,\mu'_i} \left\langle M'j; \tilde{\mathbf{P}}'\mu' \mid \tilde{\mathbf{p}}'_1\mu'_1\tilde{\mathbf{p}}'_2\mu'_2\tilde{\mathbf{p}}'_3\mu'_3 \right\rangle$$
$$\times \left\langle \tilde{\mathbf{p}}'_1\mu'_1\tilde{\mathbf{p}}'_2\mu'_2\tilde{\mathbf{p}}'_3\mu'_3 \right| I^+(0) \left| \tilde{\mathbf{p}}_1\mu_1\tilde{\mathbf{p}}_2\mu_2\tilde{\mathbf{p}}_3\mu_3 \right\rangle$$
$$\times \left\langle \tilde{\mathbf{p}}_1\mu_1\tilde{\mathbf{p}}_2\mu_2\tilde{\mathbf{p}}_3\mu_3 \mid Mj\tilde{\mathbf{P}}\mu \right\rangle, \tag{27}$$

where the light-front momenta satisfies $\tilde{\mathbf{P}} = \tilde{\mathbf{p}}_1 + \tilde{\mathbf{p}}_2 + \tilde{\mathbf{p}}_3$, and they are related to the set of coordinates $\{\mathbf{k}_1, \mathbf{k}_1, \mathbf{k}_3\}$ in the baryon rest frame through the following transformation

$$x_i = p_i^+/P^+,$$

$$\mathbf{k}_{i\perp} = \mathbf{p}_{i\perp} - x_i \mathbf{P}_\perp,$$

$$k_{i3} = \frac{1}{2}\left[x_i M_0 - \frac{m_i^2 + \mathbf{k}_{i\perp}^2}{x_i M_0}\right],$$

with $M_0 := \omega_1 + \omega_2 + \omega_3$ and Jacobi determinant

$$\left| \frac{\partial(\tilde{\mathbf{p}}_1, \tilde{\mathbf{p}}_2, \tilde{\mathbf{p}}_3)}{\partial(\tilde{\mathbf{P}}, \mathbf{k}_1, \mathbf{k}_2)}\right| = \frac{p_1^+ p_2^+ p_3^+ M_0}{\omega_1 \omega_2 \omega_3 P^+}.$$

The nucleon state vectors are related to the wave funtions as follows:

$$\begin{aligned}
\left\langle \tilde{\mathbf{p}}_1 \mu_1 \tilde{\mathbf{p}}_2 \mu_2 \tilde{\mathbf{p}}_3 \mu_4 \mid Mj; \tilde{\mathbf{P}}\mu \right\rangle &= (2\pi)^3 \delta(\tilde{\mathbf{p}}_1 + \tilde{\mathbf{p}}_2 + \tilde{\mathbf{p}}_3 - \tilde{\mathbf{P}}) \left| \frac{\partial(\tilde{\mathbf{p}}_1, \tilde{\mathbf{p}}_2, \tilde{\mathbf{p}}_3)}{\partial(\tilde{\mathbf{P}}, \mathbf{k}_1, \mathbf{k}_2)}\right|^{-1/2} \\
&\quad \times \left\langle \tfrac{1}{2}\bar{\mu}_1 \tfrac{1}{2}\bar{\mu}_2 \mid s_{12}\mu_{12}\right\rangle \left\langle s_{12}\mu_{12} \tfrac{1}{2}\bar{\mu}_3 \mid s\mu_s \right\rangle \\
&\quad \times \left\langle l_\xi \mu_\xi l_\eta \mu_\eta \mid L\mu_L\right\rangle \left\langle L\mu_L s\mu_s \mid j\mu\right\rangle \\
&\quad \times Y_{l_\xi \mu_\xi}(\hat{\mathbf{k}}_\xi) Y_{l_\eta \mu_\eta}(\hat{\mathbf{k}}_\eta) \Psi_M^{(R)j}(|\mathbf{k}_\xi|, |\mathbf{k}_\eta|) \\
&\quad \times D_{\bar{\mu}_1 \mu_1}^{(1/2)\dagger}[R_{cf}(k_1)] D_{\bar{\mu}_2 \mu_2}^{(1/2)\dagger}[R_{cf}(k_2)] \\
&\quad \times D_{\bar{\mu}_3 \mu_3}^{(1/2)\dagger}[R_{cf}(k_3)], \quad (28)
\end{aligned}$$

with the radial part $\Psi_M^{(R)j}$ of the wave funtion defined in (4), and the $SU(2)$ representation $D_{\bar{\mu}_i \mu_i}^{(1/2)}[R_{cf}(k_i)]$ for the Melosh rotation [30].

In the impulse approximation, the three-quark current operator is written as a sum of three single-quark current operators. Therefore, the current matrix element between two nucleon states (27) in this approximation is given by

$$\begin{aligned}
\left\langle M'j; \tilde{\mathbf{P}}'\mu' \middle| I^+(0) \middle| Mj\tilde{\mathbf{P}}\mu \right\rangle &= \int \frac{d\tilde{\mathbf{p}}_1}{(2\pi)^3} \frac{d\tilde{\mathbf{p}}_2}{(2\pi)^3} \frac{d\tilde{\mathbf{p}}_3}{(2\pi)^3} \int \frac{d\tilde{\mathbf{p}}'_1}{(2\pi)^3} \sum_{\mu_i, \mu'_1} \left\langle M'j; \tilde{\mathbf{P}}'\mu' \mid \tilde{\mathbf{p}}_1 + \tilde{\mathbf{q}}\mu'_1 \tilde{\mathbf{p}}_2 \mu_2 \tilde{\mathbf{p}}_3 \mu_3 \right\rangle \\
&\quad \times \left\langle \tilde{\mathbf{p}}_1 + \tilde{\mathbf{q}}\mu'_1 \middle| I_1^+(0) \middle| \tilde{\mathbf{p}}_1 \mu_1 \right\rangle \left\langle \tilde{\mathbf{p}}_1 \mu_1 \tilde{\mathbf{p}}_2 \mu_2 \tilde{\mathbf{p}}_3 \mu_3 \mid Mj\tilde{\mathbf{P}}\mu \right\rangle \\
&\quad + \text{analog terms for } \tilde{\mathbf{p}}_2 \text{ and } \tilde{\mathbf{p}}_3. \quad (29)
\end{aligned}$$

We prefer to use in this initial investigations, with the 't Hooft interaction, the most simple *Ansatz* for the single-particle current, namely to consider only the constituent quark charge operators without anomalous constituent quark magnetic moments, i.e.

$$\left\langle \tilde{\mathbf{p}}'\mu' \middle| I_i^+(0) \middle| \tilde{\mathbf{p}}\mu \right\rangle = \delta_{\mu'\mu} \hat{e}_i, \quad (30)$$

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where $\hat{e}_i$ represents the quark charge operator.

All calculations has been performed up to 6 harmonic-oscillator shells. The multidimensional integrals have been calculated with a *VEGAS* integration routine [31]. In order to examine the relevance of the components of the wave funtion to the form factors, we calculate first the proton and neutron form factors taking different number of components. In Fig. 2 the curve (a) represents the proton electric form factor G_E^p with Model 1, and the curves (b) and (c) represent the same calculation, but taking into account only the largest and the two largest components of the proton wave funtion respectively. One sees that the calculation with only 2 components almost represent the full result. In the case of the neutron the situation is a little different. We show in Fig. 3 the neutron electric form factor G_E^n with Model 1. The curve (a) represents full result, and the curves (b), (c) and (d) represent the results taking into account only the largest, the two largest and the three largest components of the neutron wave funtion respectively. We observe that just after considering the third component of the wave funtion the form factor become positive. In fact, the first two components are S-waves, and particularly in the neutron case the inclusion of other wave types seems to play an important role.

We present in Fig. 4 and in Fig. 5 the proton electric form factor G_E^p and magnetic form factor G_M^p (in μ_N) respectively. Our calculations are compared with experimental data taken from Ref. [32, 33]. The oscillator parameter β has been chosen in such a way as to reproduce the proton electric form factor. Its value is about 0.5 fm in both models and it stay in the region of the stable solutions. In order to investigate the effect of the 't Hooft interaction, we switched off the strength constant g and calculated again the form factors using the Model 1 and the same scale parameter β. In Fig. 4 and Fig. 5 the curves (a) and (b) are obtained with Models 1 and 2 respectively, and the curves (c) represent the results with Model 1 without the 't Hooft interaction.

The same form factors are presented in Fig. 6 and in Fig. 7 for the neutron. Experimental data are taken from Ref. [1, 3, 34-36]. The curves (a) and (b) are again obtained with Models 1 and 2, respectively, and the curves (c) represent the results with Model 1 without 't Hooft

interaction. We note that the result for the neutron electric form factor is the most sensitive to the 't Hooft interaction. Without this interaction the neutron wave funtion contains almost just S-waves, and the form factor behaves like in Fig. 3 (curves (b) and (c)). The 't Hooft interaction induces some configuration mixing due to diquark correlations. We did not know much about the effects from this interaction on form factors up to now. What we see from the present model is a quite interesting result, namely the configuration mixing yields the correct sign of the neutron electric form factor G_E^n.

IV. SUMMARY AND OUTLOOK

A phenomenological relativistic constituent quark model, which includes instanton-induced forces has been introduced to evaluate the electromagnetic nucleon form factors. We have performed our calculation within a light-front RHD in the impulse approximation. The internal baryon dynamics have been identified with the Salpeter Hamiltonian. It is composed of a relativistic kinetic energy part, a confinement potential and a residual 't Hooft interaction. With five parameters, namely the nonstrange quark mass m_n, the constant a and the slope b of the confinement potential, the coupling constant g and the effective range λ of the 't Hooft interaction (Tab. I), we have obtained a very good description of the proton electric form factor G_E^p up to $Q^2 = 3$ GeV^2. The other nucleon form factors are reasonably described.

In spite of considering no constituent quark form factor, the magnetic moments come out only about 10% too small for the proton and about 15% too small for the neutron. A general better description of the magnetic form factors can be expected with the consideration of constituent quark form factors (see e.g. Ref. [9, 11]). Calculations beyond the impulse approximation should also improve the results. They are in general very involved and rarely discussed in the literature.

We have examined the effects of the 't Hooft interaction on the nucleon form factors. It has been shown that, if this interaction is switched off, our results are very similar to the

results with a symmetric wave. We have suggested, that the diquark correlations generated by the 't Hooft interaction represent a very important component for the description of the neutron electric form factor G_E^n.

It should also be interesting to extended the investigations presented here to compute other baryon dynamical observables of great interest, like the proton axial-vector form factor, the electroexcitation helicity amplitudes of the nucleon- and the Δ-resonances, the proton polarizabilities, and the form factors in the strange sectors.

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TABLES

Parameter	Model 1	Model 2
m_n	220 MeV	270 MeV
a	-640 MeV	-512 MeV
b	688 MeV/fm	526 MeV/fm
g	118 MeV fm^3	118 MeV fm^3
λ	0.333 fm	0.333 fm

TABLE I. Set of parameters in Models 1 and 2.

Baryon	Model 1	Model 2	Experiment
$N(\frac{1}{2}^+, 939)$	934	935	$938 - 939$
$N(\frac{1}{2}^+, 1440)$	1556	1569	$1430 - 1470$
$N(\frac{1}{2}^-, 1535)$	1403	1404	$1520 - 1555$
$\Delta(\frac{3}{2}^+, 1232)$	1221	1229	$1229 - 1235$
$\Delta(\frac{7}{2}^+, 1950)$	1852	1835	$1935 - 1965$
$\Delta(\frac{11}{2}^+, 2420)$	2259	2240	$2300 - 2500$

TABLE II. Baryon mass spectrum in Models 1 and 2 (in MeV). Experimental data are from Ref. [29].

FIGURE CAPTIONS

FIG. 1. Diagrammatic representation of the Salpeter kernel K, with the two- and three-body kernels $K^{(2)}$ and $K^{(3)}$ respectively. We approximate K by the collective instantaneous kernel V.

FIG. 2. Proton electric form factor G_E^p with Model 1 (a). The curves (b) and (c) represent the results taking into account the largest and the two largest components of the proton wave function respectively.

FIG. 3. Neutron electric form factor G_E^n with Model 1 (a). The curves (b), (c) and (d) represent the results taking into account only the largest, the two largest, and the three largest components of the neutron wave function respectively.

FIG. 4. Proton electric form factor G_E^p. The curves (a) and (b) are obtained with Models 1 and 2 respectively. The curve (c) represents the results with Model 1 taking into account only the confinement potential, i.e. without 't Hooft interaction. The experimental results have been taken from Ref. [32, 33] .

FIG. 5. Proton magnetic form factor G_M^p (in μ_N); key as in Fig. 3. Experimental data are from Ref. [33].

FIG. 6. Neutron electric form factor G_E^n; key as in Fig. 3. Experimental data are from Ref. [34, 35].

FIG. 7. Neutron magnetic form factor G_M^n (in μ_N); key as in Fig. 3. Experimental data are from [1, 3, 36].

FIGURE 1
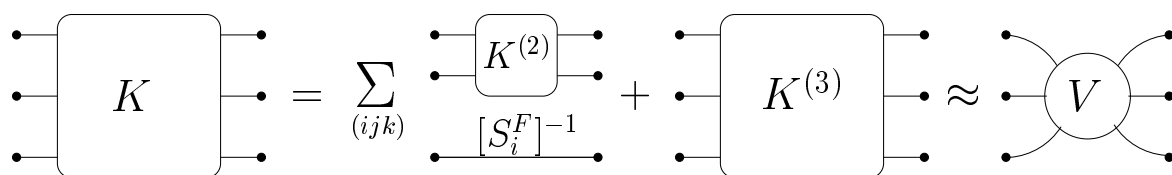

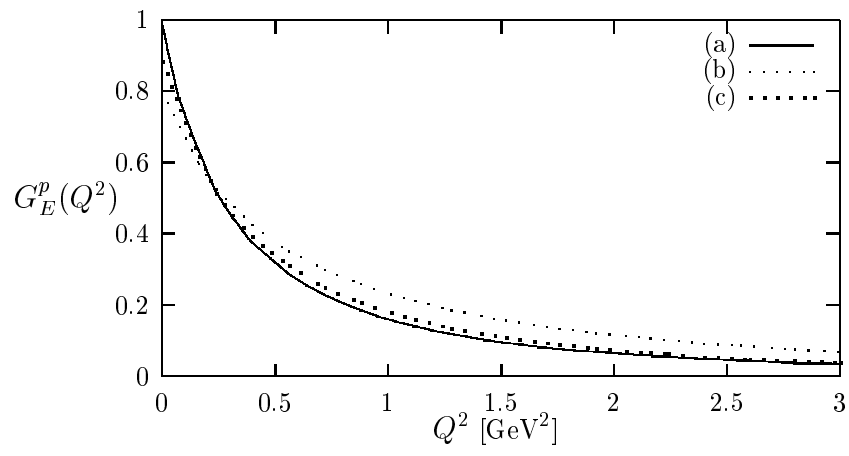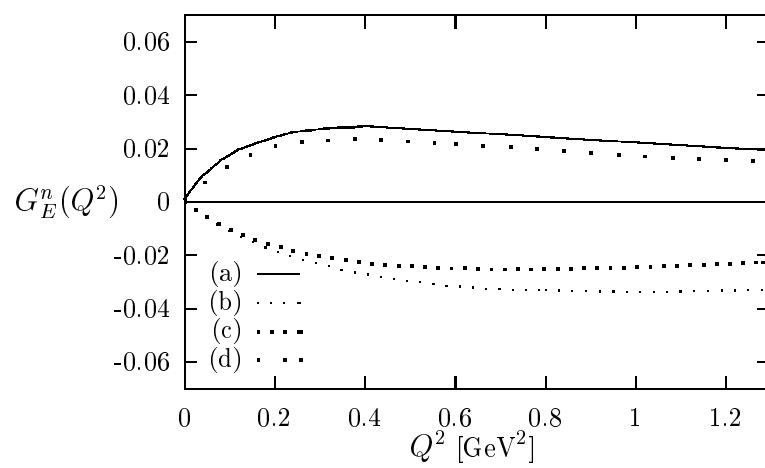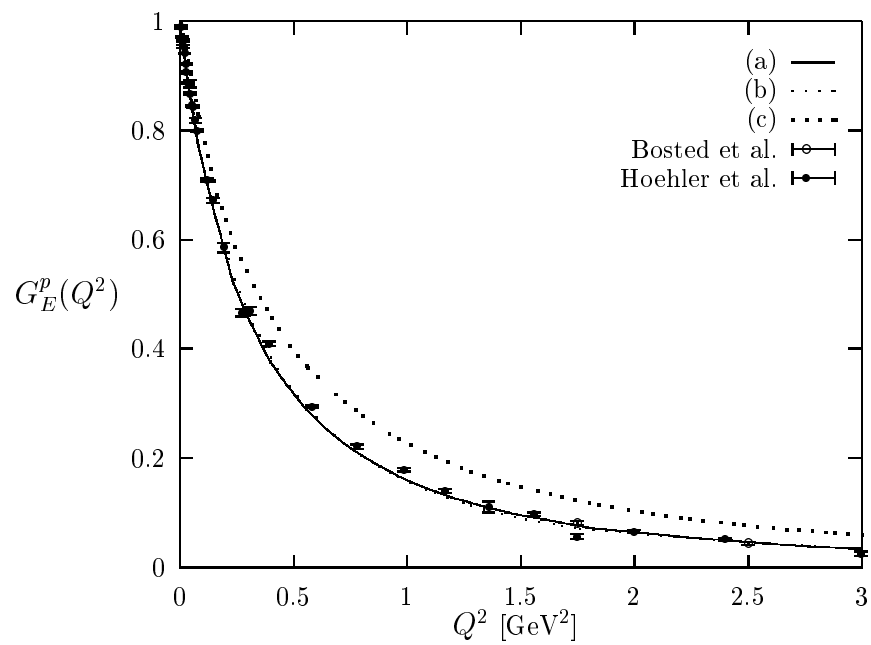$$K = \sum_{(ijk)} \begin{array}{c} K^{(2)} \\ [S_i^F]^{-1} \end{array} + K^{(3)} \approx V$$

FIGURE 2

FIGURE 3

FIGURE 4

FIGURE 5

FIGURE 6

The figure plots $G_E^n(Q^2)$ versus Q^2 [GeV^2]. Curves labeled (a) solid, (b) dotted, (c) heavy dotted. Data points from Simon et al. and Platchkov et al.

FIGURE 7